NAG C Library Function Document

nag_prob_non_central_students_t (g01gbc)

1 Purpose

nag_prob_non_central_students_t (g01gbc) returns the lower tail probability for the non-central Student's t-distribution.

2 Specification

```
#include <nag.h>
#include <nagg01.h>

double nag_prob_non_central_students_t (double t, double df, double delta,
          double tol, Integer max_iter, NagError *fail)
```

3 Description

The lower tail probability of the non-central Student's t-distribution with ν degrees of freedom and non-centrality parameter δ, $P(T \leq t : \nu; \delta)$ is defined by:

$$P(T \leq t : \nu; \delta) = C_\nu \int_0^\infty \left(\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha u - \delta} e^{-x^2/2}\, dx \right) u^{\nu-1} e^{-u^2/2} du, \quad \nu > 0.0$$

with

$$C_\nu = \frac{1}{\Gamma(\frac{1}{2}\nu) 2^{(\nu-2)/2}}, \quad \alpha = \frac{t}{\sqrt{\nu}}$$

The probability is computed in one of two ways,

(a) when $t = 0.0$, the relationship to the normal is used

$$P(T \leq t : \nu; \delta) = \frac{1}{\sqrt{2\pi}} \int_\delta^\infty e^{-u^2/2} du;$$

(b) otherwise the series expansion described in Amos (1964) (equation 9) is used. This involves the sums of confluent hypergeometric functions, the terms of which are computed using recurrence relationships.

4 Parameters

1: **t** – double *Input*

On entry: the deviate from the Student's t-distribution with ν degrees of freedom, t.

2: **df** – double *Input*

On entry: the degrees of freedom of the Student's t-distribution, ν.

Constraint: **df** $\geq$ 1.0.

3: **delta** – double *Input*

On entry: the non-centrality parameter of the Students t-distribution, δ.

4: **tol** – double *Input*

On entry: the absolute accuracy required by the user in the results.
If nag_prob_non_central_students_t is entered with **tol** greater than or equal to 1.0 or less than $10 \times$***machine precision*** (see nag_machine_precision (X02AJC)), then the value of $10 \times$***machine precision*** is used instead.

5: **max_iter** – Integer *Input*

On entry: the maximum number of terms that are used in each of the summations.

Suggested value: 100. See Section 6 for further comments.

Constraint: **max_iter** ≥ 1.

6: **fail** – NagError * *Input/Output*

The NAG error parameter (see the Essential Introduction).

5 Error Indicators and Warnings

NE_REAL_ARG_LT

On entry, **df** must not be less than 1.0: **df** = <*value*>.

NE_INT_ARG_LT

On entry, **max_iter** must not be less than 1: **max_iter** = <*value*>.

NE_SERIES

One of the series has failed to converge with **df** = <*value*> and **max_iter** = <*value*>. Reconsider the requested tolerance and/or the maximum number of iterations.

NE_PROBABILITY

The probability is too small to calculate accurately.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

6 Further Comments

The rate of convergence of the series depends, in part, on the quantity: $t^2/(t^2+\nu)$. The smaller this quantity the faster the convergence. Thus for large t and small ν the convergence may be slow. If ν is an integer then one of the series to be summed is of finite length.

If two tail probabilities are required then the relationship of the t-distribution to the F-distribution can be used:

$$F = T^2,\ \lambda = \delta^2,\ \nu_1 = 1 \quad \text{and} \quad \nu_2 = \nu,$$

and a call made to nag_prob_non_central_f_dist (g01gdc).

Note: this routine only allows degrees of freedom greater than or equal to 1 although values between 0 and 1 are theoretically possible.

6.1 Accuracy

The series described in Amos (1964) are summed until an estimated upper bound on the contribution of future terms to the probability is less than **tol**. There may also be some slight loss of accuracy due to calculation of gamma functions. For large values of $\delta > 50$ there may be significant loss of accuracy.

6.2 References

Amos D E (1964) Representations of the central and non-central t-distributions *Biometrika* **51** 451–458

7 See Also

nag_prob_non_central_students_t (g01gbc)

8 Example

Values from, and degrees of freedom for and non-centrality parameter of the non-central Student's t-distributions are read, the lower tail probabilities calculated and all these values printed until the end of data is reached.

8.1 Program Text

```
/* nag_prob_non_central_students_t (g01gbc) Example Program.
 *
 * Copyright 1999 Numerical Algorithms Group.
 *
 * Mark 6, 2000.
*/

#include <stdio.h>
#include <nag.h>
#include <nagg01.h>

int main(void)
{
  double delta, df, prob, t, tol;
  Integer max_iter;
  Integer exit_status = 0;
  NagError fail;

  INIT_FAIL(fail);
  Vprintf("g01gbc Example Program Results\n\n");

  /*  Skip heading in data file */
  Vscanf("%*[^\n]");

  Vprintf("      t       df    delta    prob\n\n");
  tol = 5e-6;
  max_iter = 50;
  while ((scanf("%lf %lf %lf %*[^\n]", &t, &df, &delta)) != EOF)
    {
      prob = g01gbc(t, df, delta, tol, max_iter, &fail);
      if (fail.code == NE_NOERROR)
 Vprintf(" %8.3f%8.3f%8.3f%8.4f\n", t, df, delta, prob);
      else
 {
   Vprintf("Error from g01gbc.\n%s\n", fail.message);
   exit_status=1;
          goto END;
 }
    }
END:
  return exit_status;
}
```

8.2 Program Data

```
g01gbc Example Program Data
 -1.528  20.0  2.0           :t df delta
 -0.188   7.5  1.0           :t df delta
  1.138  45.0  0.0           :t df delta
```

8.3 Program Results

```
g01gbc Example Program Results

      t       df    delta    prob

   -1.528  20.000   2.000  0.0003
   -0.188   7.500   1.000  0.1189
    1.138  45.000   0.000  0.8694
```